

FOR OFFICIAL USE

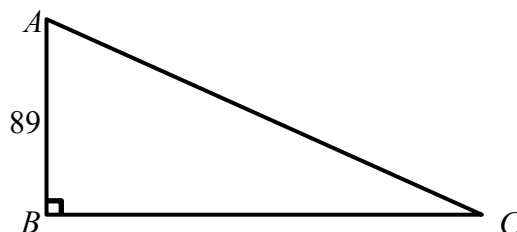
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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特別聲明，答案須用數字表達，並化至最簡。

1. Find the perimeter P of the right-angled triangle ABC if all the side lengths are positive integers and $AB = 89$.

若直角三角形 ABC 所有邊長為正整數，且 $AB = 89$ ，求三角形 ABC 的周界 P 。



2. If A is the units digit of $8888^{20242024}$, find the value of A .

若 A 是 $8888^{20242024}$ 的個位數，求 A 的值。

3. How many 5-digit numbers contain at least one “1” and at least one “3”?

有多少個 5 位數包含最少 1 個「1」和最少 1 個「3」？

4. Let m be the number of possible pairs of positive integers a and b for which $a^4 + 4b^4$ is a prime number. Find m .

設有 m 對正整數 a 和 b ，使 $a^4 + 4b^4$ 為質數，求 m 的值。

Hong Kong Mathematics Olympiad (2023/24)
Finals (Group – Event 2)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Let $x > 0$. Given that $x - \frac{1}{x} = \sqrt{3}$ and $a = x^5 + x^3 + x + \frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^5}$, find the value of a .

設 $x > 0$ 。已知 $x - \frac{1}{x} = \sqrt{3}$ 且 $a = x^5 + x^3 + x + \frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^5}$ ，求 a 的值。

$a =$

2. Using the first 2024 positive integers: 1, 2, 3, 4, 5, 6, ..., 2024, a new integer is formed as 123456789101112...2024. If b is the number of “0” in this integer, find the value of b .

用首 2024 個正整數：1, 2, 3, 4, 5, 6, ..., 2024, 造出一個新的整數：123456789101112...2024。若 b 是這個整數裡「0」的數量，求 b 的值。

$b =$

3. c is the number of positive factors of $2024^2 - 2023^2$. Find the value of c .

c 是 $2024^2 - 2023^2$ 的正因數的數量。求 c 的值。

$c =$

4. Let “0”, “1”, “2”,, “6” represent Sunday, Monday, Tuesday, and Saturday, respectively. Today is Monday. If “ d ” represents the day of week that comes after $20^{24^{2024}}$ days. Find d .

假設「0」、「1」、「2」、.....、「6」分別為星期日、星期一、星期二、.....和星期六。今日是星期一，若 $20^{24^{2024}}$ 天後的那一天是星期幾之代號為「 d 」，求 d 。

$d =$

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Find the smallest positive integer n such that $2^{10} + 2^{13} + 2^n$ is a perfect square number.

試找出最小的正整數 n 使得 $2^{10} + 2^{13} + 2^n$ 成為一完全平方數。

2. Suppose $a^2 + b^2 + 6a - 14b + 58 = 0$. Find $b - a$.

設 $a^2 + b^2 + 6a - 14b + 58 = 0$ 。求 $b - a$ 。

3. There was a chest containing \$8,000 buried in one of the corners of a square piece of land. In a contest, you and another man called “Mr. Badluck” were digging for the chest. Mr. Badluck had one peculiarity: he always made the wrong choice. You won the toss and chose first. You picked a corner, and Mr. Badluck picked another. Before you started, you observed that Mr. Badluck found no chest. The rules of the game allowed you to make a switch to another corner, but with a penalty of \$200. Should you make a switch? Calculate the expected gain from making the switch in dollars.

在正方形土地的某一個角落裡埋著一個裝有 \$8,000 的箱子。在一次比賽中，你和另一個叫「倒霉先生」的人一起挖箱子。倒霉先生有一個特點：他總是做出錯誤的選擇。你贏了擲骰子先選。你選了一個角落，倒霉先生選了另一個角落。在你準備開始時，你發現倒霉先生沒有找到箱子。遊戲規則允許你換另一個角落，但要罰 \$200。你應否更換嗎？計算換角落的期望收益。

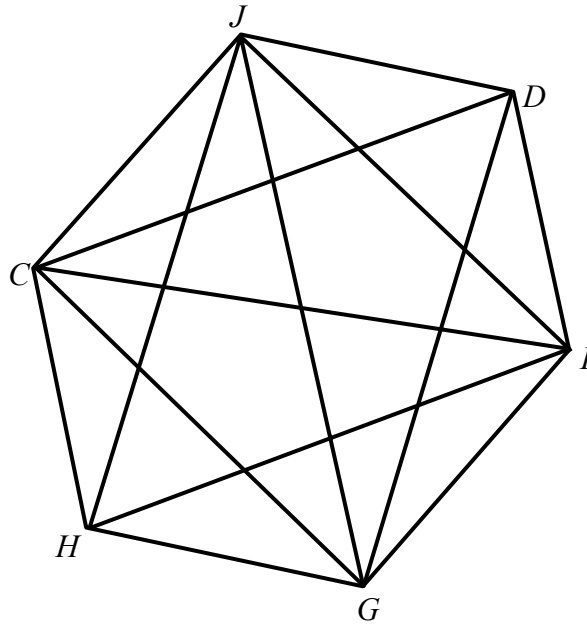
4. A convex hexagon has the following property:
- (i) all the triangles formed from any vertex with the two adjacent vertices have an area of $1,000 \text{ cm}^2$; and
 - (ii) $CH = DI$.

What is the area of the hexagon?

一個凸六邊形有以下性質：

- (i) 由任意頂點與相鄰兩個頂點組成的三角形的面積都是 $1,000 \text{ cm}^2$ ；及
- (ii) $CH = DI$ 。

求六邊形的面積。



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1. Let a, b be non-zero integers satisfying the equation $a - ab + b = 18$. Find $a + b$.

設 a, b 為非零整數，且滿足方程 $a - ab + b = 18$ 。求 $a + b$ 。

$a + b =$

2. Let x be a positive integer satisfying $x(x+1)(x+2)(x+3) = 3024$. Find x .

設 x 為一正整數，且滿足 $x(x+1)(x+2)(x+3) = 3024$ 。求 x 。

$x =$

3. Let α, β be the two roots of the quadratic equation $x^2 + 6x + 2 = 0$. Find the quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$, and coefficient of x^2 is 1.

設 α, β 為二次方程 $x^2 + 6x + 2 = 0$ 的兩個根，求以 $\frac{\alpha^2}{\beta}$ 和 $\frac{\beta^2}{\alpha}$ 為根及 x^2 的系數為 1 的二次方程。

4. The unshaded part in the diagram below is made up of a quarter-circle and a semi-circle. Find the area of the shaded part.

下圖空白部分由一個四分之一圓和一個半圓組成，求陰影部分的面積。

